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The Goos-Hänchen Shift on a Layer

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Abstract

A beam of linearly polarised light suffering total internal reflection is shifted longitudinally. Two different shifts are found, belonging to the eigenstates of TE and TM polarisation. A much smaller transverse shift occurs when left and right circular polarisation are considered as eigenstates. The measured shifts agree well with theoretical predictions and are referred to as Goos–Hänchen effect. In this contribution shifts are investigated for the case of frustrated total internal reflection on a thin optical barrier. As well known, there appears, in addition to the reflected beam, a transmitted beam for which hitherto the shifts were not reported.

1. *Introduction*

The shifts of a light beam suffering total internal reflection, or the Goos-Hänchen effect, recently have attracted appreciable attention. The longitudinal shift of an unpolarised beam was measured exploiting repeated total reflection in a glass plate (Goos & Hänchen, 1947). Apparently, some authors were aware of this effect long ago. Calculations indicated that the shift of a linear polarised beam should depend on the direction of polarisation (Artmann, 1948; Fragstein, 1949). This was confirmed experimentally (Goos & Lindberg-Hänchen, 1949). Reliable measurements gave full support to results of improved calculations (Wolter, 1950). Formulas, valid also for angles of incidence far away of the critical angle, were derived by means of the energy flux in the evanescent wave (Renard, 1964). Insignificantly different formulas were deduced directly from Maxwell's equations (Lotsch, 1968, 1970, 1971).†

Experiments with laser light gave new impetus to the field (Mazet *et al.*, 1971). They demonstrated that in observing the longitudinal shift an unpolarised beam is split into two, corresponding to the eigenstates TE and TM. This

† Lotsch's published thesis (1970, 1971) contains an elaborate list of references.

was interpreted as a Stern–Gerlach type of experiment with photons and it was speculated that the photon may have a finite rest mass (de Broglie & Vigier, 1972). The transverse shift was measured (Imbert, 1970, 1972a, 1972b) being predicted some time ago (de Beauregard, 1965; Schilling, 1965).

From a theoretical point of view the evanescent wave in optically less-dense medium offers an interesting possibility: it may have some tachyonic properties (de Beauregard *et al.*, 1971; de Beauregard, 1973).† It is understood that within the evanescent wave signals cannot be transmitted faster than light *in vacuo*, since on the contrary macrocausality would be violated. The expected tachyonic properties can only be of local character. They may show up in interaction of photons in the evanescent wave with an atomic system. The component of the momentum $\hbar q$ transferred to the system and the corresponding transferred energy $\hbar\omega$ may obey in this case a relation $\hbar\omega/\hbar q > c$, c being the velocity of light *in vacuo.* Carniglia & Mandel (1971) have quantised the evanescent electromagnetic field and claim that the incident, evanescent, and reflected fields comprise an entirety. This would entail that the tachyonic character of evanescent photons may not be directly observable. Conclusive experiments, as yet, do not exist. Thus, the observability of the local tachyonic character of evanescent photons is still questionable.

In view of these interesting implications it appears worthwhile to investigate all possible aspects of the Goos–Hänchen effect and of the evanescent wave. Such reasoning lead us to the investigation of the Goos–Hänchen effect in frustrated total reflection. We examined the field in the evanescent wave and calculated the shifts of the reflected and transmitted beam considering a thin layer of optically less-dense medium. Section 2 deals with electromagnetic field within and outside the layer in the plane wave approximation. Boundary conditions are exploited to get the amplitudes of fields. In Section 3 the longitudinal shifts of the reflected and transmitted beam are deduced. This is done for linear polarisation first by means of a simple spatially modulated plane wave. In Section 4 arguments concerning the energy flux in the evanescent wave are exploited. The transverse shifts are calculated by the same argument in Section 5. In Section 6 the main results of the calculations are discussed and the feasibility of experimental observation is considered.

2. *Reflection and Transmission Coefficients*

In order that the beam of electromagnetic waves is well defined its width has to be of the order of tens of wavelengths whereas the layer thickness, in the range of interest, has the order of magnitude of a wavelength. Thus, in our considerations the plane wave approximation is justified.

The layer of thickness Z and index of refraction $n_r = \epsilon_r^{1/2}$ is bounded by planes $z = 0$ and $z = Z$. It is surrounded on both sides by a medium with index of refraction $n_i = \epsilon_i^{1/2}$. The magnetic permeabilities of both media, μ_r and μ_i,

† The articles of de Beauregard *et al.* (1971) and of de Beauregard (1973) present a thorough review, whereas we quote the main contributions only.

are put equal to unity. The plane xz is the plane of incidence and θ_i is the angle of incidence.

A solution of Maxwell's equations, involving an incident wave, can be constructed in the following way:

$z<0$

$$\mathbf{E}_1 = \exp(-i\omega t)\{(-E_\parallel \cos\theta_i, E_\perp, E_\parallel \sin\theta_i)\exp[in_ik_0(x\sin\theta_i + z\cos\theta_i)] + (R_\parallel E_\parallel \cos\theta_i, R_\perp E_\perp, R_\parallel E_\parallel \sin\theta_i)\exp[in_ik_0(x\sin\theta_i - z\cos\theta_i)]\}$$

$0<z<Z$

$$\mathbf{E}_2 = \exp(-i\omega t)\{(-S_\parallel E_\parallel \cos\theta_r, S_\perp E_\perp, S_\parallel E_\parallel \sin\theta_r)\exp[in_rk_0(x\sin\theta_r + z\cos\theta_r)] + (P_\parallel E_\parallel \cos\theta_r, P_\perp E_\perp, P_\parallel E_\parallel \sin\theta_r)\exp[in_rk_0(x\sin\theta_r - z\cos\theta_r)]\}$$

$Z<z$

$$\mathbf{E}_3 = \exp(-i\omega t)(-T_\parallel E_\parallel \cos\theta_i, T_\perp E_\perp, T_\parallel E_\parallel \sin\theta_i)\exp[in_ik_0(x\sin\theta_i + z\cos\theta_i)] \tag{2.1}$$

$k_0 = 2\pi/\lambda_0$ is the magnitude of the wave vector of the incident wave as measured in vacuum. $E_\perp$ is the component of the electric field perpendicular to the plane of incidence and $E_\parallel$ is the component parallel to the plane of incidence. The components $E_\perp$ and $E_\parallel$ in the incident wave are connected with the corresponding components in the reflected wave through the reflection coefficients $R_\perp$ and $R_\parallel$ and with those in the transmitted wave through the transmission coefficients $T_\perp$ and $T_\parallel$. The coefficients $S_\perp, S_\parallel, P_\perp$, and $P_\parallel$ connect the field in the incident wave with the field inside the layer. The angle θ_r is given by Snell's law $n_i \sin\theta_i = n_r \sin\theta_r$.

The magnetic fields in the above plane waves are obtained as $\mathbf{B} = nc^{-1}(\mathbf{k}/k) \times \mathbf{E}$, with the corresponding wave vector $\mathbf{k}$.

The fields in the layer and in the surrounding medium are coupled together with the well-known boundary conditions: the tangential components of electric field $\mathbf{E}$, the normal components of $\epsilon\mathbf{E}$ and all the components of the magnetic field $\mathbf{B}$ are to be continuous across both boundary planes. Eight independent equations, obtained from the above conditions, decouple into two distinct sets corresponding to the TE or $\perp$ and to the TM or $\parallel$ polarisation (Kodre & Strnad, 1973). Solving these equations for the coefficients R, S, P, and T we obtain

$$\begin{aligned} R &= \alpha^{-1}\beta_1 \sin\delta_r \\ S &= \alpha^{-1}\beta_2\, e^{-i\delta_r} n_i \cos\theta_i \\ P &= \alpha^{-1}\beta_3\, e^{i\delta_r} n_i \cos\theta_i \\ T &= 2\alpha^{-1} n_i n_r \cos\theta_i \cos\theta_r\, e^{-i\delta_i} \end{aligned} \tag{2.2}$$

Shorthand notations $\delta_i = n_i k_0 Z \cos\theta_i$ and $\delta_r = n_r k_0 Z \cos\theta_r$ and

$$\begin{aligned} \alpha_\perp &= 2n_i n_r \cos\theta_i \cos\theta_r \cos\delta_r - i(n_i^2 \cos^2\theta_i + n_r^2\cos^2\theta_r)\sin\delta_r \\ \beta_{1\perp} &= n_i^2\cos^2\theta_i - n_r^2\cos^2\theta_r \\ \beta_{2\perp} &= n_r\cos\theta_r + n_i\cos\theta_i \\ \beta_{3\perp} &= n_r\cos\theta_r - n_i\cos\theta_i \end{aligned} \tag{2.3}$$

for the TE case and

$$\begin{aligned} \alpha_\parallel &= 2n_i n_r \cos\theta_i \cos\theta_r \cos\delta_r - i(n_r^2 \cos^2\theta_i + n_i^2\cos^2\theta_r)\sin\delta_r \\ \beta_{1\parallel} &= n_r^2\cos^2\theta_i - n_i^2\cos^2\theta_r \\ \beta_{2\parallel} &= n_i\cos\theta_r + n_r\cos\theta_i \\ \beta_{3\parallel} &= n_i\cos\theta_r - n_r\cos\theta_i \end{aligned} \tag{2.4}$$

for the TM case were introduced. The coefficients R and T are often encountered in the literature (e.g. Hall, 1902).

3. *The Longitudinal Shift*

Let A denote the amplitude of the transverse field (E in the TE case, B in the TM case). RA and TA are then the amplitudes of the reflected and transmitted wave, respectively, provided that the proper index $\perp$ or $\parallel$ is taken for R and T. Following Wolter (1950), let us further consider a compositum of two plane waves with the same frequency and amplitude but slightly different incident angles θ_i and $\theta_i - d\theta_i$, respectively. If a phase difference of π is introduced the compositum

$$\begin{aligned} &(0, A\exp[in_i k_0(x\sin\theta_i + z\cos\theta_i)] \\ &-A\exp[in_i k_0\{x\sin(\theta_i + d\theta_i) - z\cos(\theta_i - d\theta_i)\}], 0) \\ &= (0, \frac{\partial}{\partial\theta_i}\{A\exp[in_i k_0(x\sin\theta_i + z\cos\theta_i)]\}, 0)\,d\theta_i \end{aligned} \tag{3.1}$$

will be marked by a plane $x\cos\theta_i - z\sin\theta_i = 0$ of zero field. Similarly, the reflected wave compositum can be written as

$$(0, \frac{\partial}{\partial\theta_i}\{RA\exp[in_i k_0(x\sin\theta_i + z\cos\theta_i)]\}, 0)\,d\theta_i \tag{3.2}$$

Searching for the marking plane in this case, we obtain the following spatial dependence of the amplitude

$$in_i k_0 R(x\sin\theta_i + z\cos\theta_i) + \partial R/\partial\theta_i$$

Owing to the complex nature of R there is no plane of zero field. However, a plane of minimum field

$$x\sin\theta_i + z\cos\theta_i + (n_i k_0)^{-1}\,\mathrm{Im}\,(\partial R/R\partial\theta_i) = 0$$

can be found, where only the out-of-phase field $ARe\,(\partial R/R\partial\,\theta_i)$ exists. This plane is accepted as the marking plane in the reflected beam. Hence, the shift D_R can be extracted as

$$D_R = -(n_i k_0)^{-1}\,\mathrm{Im}\,(\partial R/R\,\partial\theta_i) \tag{3.3}$$

By the same device we obtain the shift in the transmitted wave as

$$D_T = -(n_i k_0)^{-1}\,\mathrm{Im}\,(\partial T/T\,\partial\theta_i) \tag{3.4}$$

Formula (3.3) was obtained following different ways by numerous authors (Artmann, 1948; Fragstein, 1949; Wolter, 1950; Chiu & Quinn, 1972) who studied the total internal reflection on a plane interface between two media.

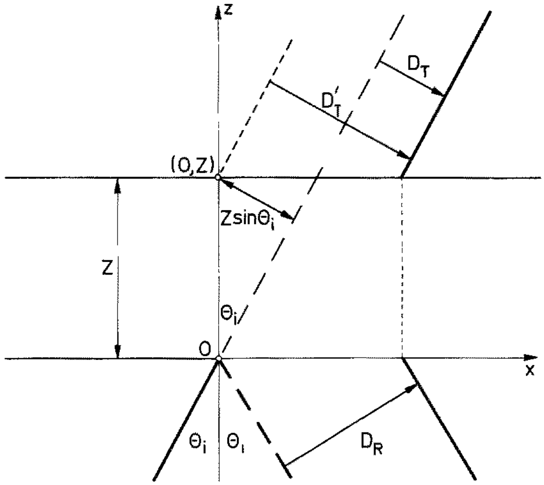

Figure 1.–Schematic presentation of the layer and the quantities involved.

Of course, $R_\perp$ and $R_\parallel$ in their calculations were the well-known Fresnel coefficients. The shifts obtained by the above method represent an approximation valid for angles of incidence close to the critical angle (Renard, 1964). Moreover, Wolter (1950) considered a more realistic case of a beam of finite angular divergence.

We shall apply the above equations to the case of reflection and transmission on a thin optical barrier called a 'totally' reflecting layer and described by coefficients (2.2)–(2.4). In this case $n_r < n_i$ and $\theta_i > \theta_c = \arcsin(n_r/n_i)$. Hence, $\cos\theta_r = i(n_i^2 \sin^2\theta_i/n_r^2 - 1)^{1/2}$ is imaginary and so is $\delta_r = n_r k_0 Z \cos\theta_r = i\gamma$ with $\gamma = k_0 Z(n_i^2 \sin^2\theta_i - n_r^2)^{1/2}$. From hereon the calculus is straightforward, though rather lengthy. A welcome general result

$$D_T = D_R + (n_i k_0)^{-1}\,\partial\delta_i/\partial\theta_i = D_R - Z\sin\theta_i \tag{3.5}$$

shows up, connecting the shifts of the reflected and transmitted beam. Led by this result we redefine the shift D_T' of the transmitted beam as a distance from the point $x = 0$, $z = Z$ rather than from the origin O (Fig. 1). Then $D_T' = D_R$, i.e. the transmitted beam lies symmetrically to the reflected one. The final result for the shift

$$D'_T = D_R = D \tag{3.6}$$

can be written in the form

$$D = D_0 F(Z) \tag{3.7}$$

Here D_0 denotes the familiar shift for a single totally reflecting plane obtained by earlier authors and corresponding to the limiting case for $Z \to \infty$ of (3.6):

$$D_{0\perp} = 2 \sin \theta_i / k_0 (n_i^2 \sin^2 \theta_i - n_r^2)^{1/2} \tag{3.8}$$

and

$$D_{0\parallel} = D_{0\perp} / \{(1 + n_i^2/n_r^2) \sin^2 \theta_i - 1\} \tag{3.9}$$

The function $F(Z)$ is

$$F(Z) = (sh\gamma ch\gamma - B)/(sh^2\gamma + C) \tag{3.10}$$

with

$$\begin{aligned} B_\perp &= n_i^2 \cos^2 \theta_i (n_i^2 + n_r^2 - 2n_i^2 \sin^2 \theta_i)/(n_i^2 - n_r^2)^2 \\ C_\perp &= 4n_i^2 \cos^2 \theta_i (n_i^2 \sin^2 \theta_i - n_r^2)/(n_i^2 - n_r^2)^2 \end{aligned} \tag{3.11}$$

and

$$\begin{aligned} B_\parallel &= n_i^2 \cos^2 \theta_i \{n_i^2 + n_r^2 - (n_r^2 + n_i^4/n_r^2) \sin^2 \theta_i\}/ \\ &\qquad (n_i^2 - n_r^2)^2 \{(1 + n_i^2/n_r^2) \sin^2 \theta_i - 1\} \\ C_\parallel &= C_\perp / \{(1 + n_i^2/n_r^2) \sin^2 \theta_i - 1\}^2 \end{aligned} \tag{3.12}$$

4. *Longitudinal Shift by the Energy Argument*

Shifts can also be deduced from the longitudinal energy flux within the layer. The flux density is the time average of the Poynting vector

$$\mathbf{j}_2 = \tfrac{1}{4} c^2 \epsilon_0 (\mathbf{E}_2 \times \mathbf{B}_2^* + \mathbf{E}_2^* \times \mathbf{B}_2) \tag{4.1}$$

Inserting the electric field $\mathbf{E}_2$ (2.1) and the corresponding magnetic field $\mathbf{B}_2$ we obtain

$$\begin{aligned} j_{2x} &= \tfrac{1}{2} c\epsilon_0 n_i \sin \theta_i \{E_\perp^* E_\perp (S_\perp^* S_\perp \,\mathrm{e}^{-2\zeta} + P_\perp^* P_\perp \,\mathrm{e}^{2\zeta} + S_\perp P_\perp^* + S_\perp^* P_\perp) \\ &\quad + E_\parallel^* E_\parallel (S_\parallel^* S_\parallel \,\mathrm{e}^{-2\zeta} + P_\parallel^* P_\parallel \,\mathrm{e}^{2\zeta} + S_\parallel P_\parallel^* + S_\parallel^* P_\parallel)\} \end{aligned} \tag{4.2a}$$

$$j_{2y} = \frac{i}{2} c\epsilon_0 (n_i/n_r)(n_i^2 \sin^2\theta_i - n_r^2)^{1/2}$$

$$\{E_\perp^* E_\parallel (S_\perp^* S_\parallel \,\mathrm{e}^{-2\zeta} - P_\perp^* P_\parallel \,\mathrm{e}^{2\zeta}) + E_\perp E_\parallel^* (P_\perp P_\parallel^* \,\mathrm{e}^{2\zeta} - S_\perp S_\parallel^* \,\mathrm{e}^{-2\zeta})\} \tag{4.2b}$$

$$j_{2z} = \frac{i}{2} c\epsilon_0 (n_i^2 \sin^2\theta_i - n_r^2)^{1/2}$$

$$\{E_\perp^* E_\perp (P_\perp^* S_\perp - P_\perp S_\perp^*) + E_\parallel^* E_\parallel (P_\parallel^* S_\parallel - P_\parallel S_\parallel^*)\} \qquad (4.2c)$$

with $\zeta = k_0(n_i^2 \sin^2\theta_i - n_r^2)^{1/2} z$.

The longitudinal component (4.2a) can be interpreted as a flow of energy giving rise to the beam shifts

$$(\tfrac{1}{2} c\epsilon_0 n_i E^* E)^{-1} \int_0^Z j_{2x}\, dz = R^* R \tilde{D}_R + T^* T \tilde{D}'_T \qquad (4.3)$$

The reflectivity R^*R and the transmissivity T^*T represent the reflected and transmitted part of the incident energy flux. For the two eigenstates TE and TM we obtain from (2.2)-(2.4) immediately

$$R^*R = sh^2\gamma/(sh^2\gamma + C) \qquad T^*T = C/(sh^2\gamma + C) \qquad (4.4)$$

The constants C are given by (3.11) and (3.12). As expected

$$R_\perp^* R_\perp + T_\perp^* T_\perp = R_\parallel^* R_\parallel + T_\parallel^* T_\parallel = 1 \qquad (4.5)$$

It is not possible to derive both shifts on the basis of the energy argument. However, we assume that equation (3.5) is valid for a wave packet of any form, though it was derived for a simple spatially modulated plane wave (3.1) only.

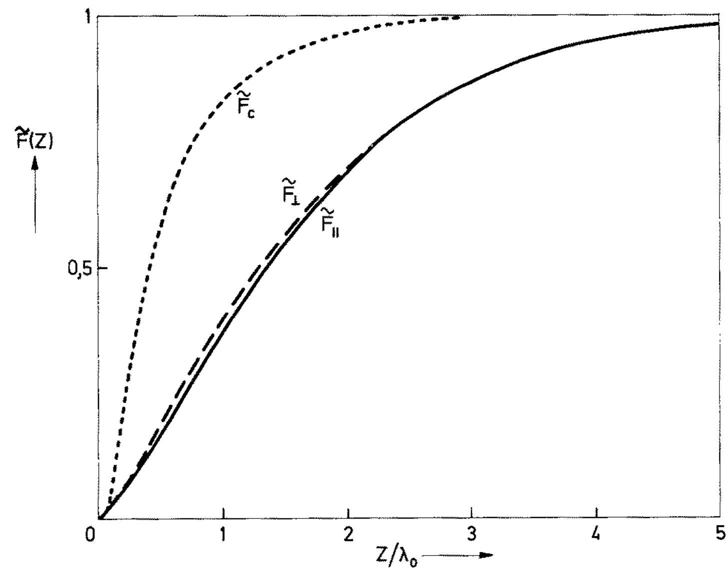

Figure 2.—Functions $\tilde{F}_\perp$, $\tilde{F}_\parallel$, and $\tilde{F}_C$ for a layer with $n_r = 1$ in glass ($n_i = 1{,}524$). The angle of incidence is $\frac{1}{4}°$ above the critical angle.

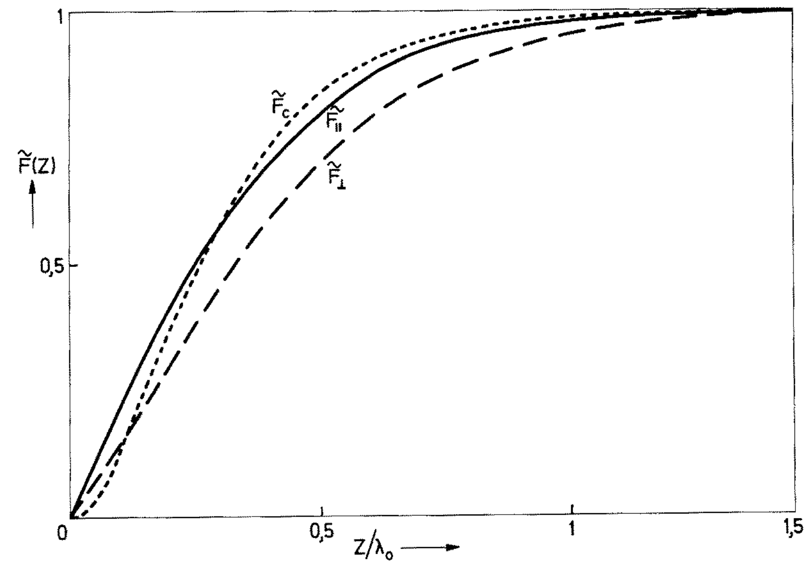

Figure 3.—Functions $\tilde{F}_\perp$, $\tilde{F}_\parallel$, and $\tilde{F}_C$ for the angle of incidence 45°. Other data as in Fig. 2.

Both shifts D_R and D_T' following from the energy argument are measured symmetrically (3.6). We put the result again in the form (3.7) with

$$\tilde{D}_{0\perp} = 2n_i^2 \sin\theta_i \cos^2\theta_i / k_0(n_i^2 \sin^2\theta_i - n_r^2)^{1/2}(n_i^2 - n_r^2) \quad (4.6)$$

the relation

$$\tilde{D}_{0\parallel} = \tilde{D}_{0\perp}/\{(1 + n_i^2/n_r^2)\sin^2\theta_i - 1\}$$

being the same as (3.9). Functions $\tilde{F}_\perp$ and $\tilde{F}_\parallel$ are of the form (3.10) with other constants B:

$$\tilde{B}_\perp = (n_i^2 + n_r^2 - 2n_i^2 \sin^2\theta_i)/(n_i^2 - n_r^2), \qquad \tilde{C}_\perp = C_\perp \quad (4.7)$$

$$\tilde{B}_\parallel = \{n_i^2 + n_r^2 - (n_r^2 + n_i^4/n_r^2)\sin^2\theta_i\}/$$
$$(n_i^2 - n_r^2)\{(1 + n_i^2/n_r^2)\sin^2\theta_i - 1\} \qquad \tilde{C}_\parallel = C_\parallel \quad (4.8)$$

Figures 2 and 3 show these functions for two particular cases. The distinction between the results of this section and the results of section 3 lies in additional factors $n_i^2 \cos^2\theta_i/(n_i^2 - n_r^2)$ which go over into unity for $\theta_i \to \theta_c$. The same factor is found to make the distinction between $\tilde{D}_{0\perp}$ and $D_{0\perp}$ and between $\tilde{D}_{0\parallel}$ and $D_{0\parallel}$ (Imbert, 1972).

5. *The Transverse Shift*

The transverse shift of circularly polarised beams is much smaller than the above longitudinal shifts and is more difficult to observe. We sketch the calculation on the basis of the energy flux, whereby we again assume that the reflected and the transmitted beam are shifted for the same amount. Analogously to equation (4.3) we obtain from the transverse component of the energy flux (4.2b)

$$\{\tfrac{1}{2}c\epsilon_0 n_i(E_\perp^* E_\perp + E_\parallel^* E_\parallel)\}^{-1}\int_0^Z j_{2y}\,dz$$

$$= \tfrac{1}{2}(R_\perp^* R_\perp + R_\parallel^* R_\parallel)\tilde{D}_{CR} + \tfrac{1}{2}(T_\perp^* T_\perp + T_\parallel^* T_\parallel)\tilde{D}_{CT} = \tilde{D}_C \qquad (5.1)$$

$\tfrac{1}{2}(R_\perp^* R_\perp + R_\parallel^* R_\parallel)$ is the reflectivity and $\tfrac{1}{2}(T_\perp^* T_\perp + T_\parallel^* T_\parallel)$ is the transmissivity for circularly polarised light. Again the result can be written as $\tilde{D}_C = \tilde{D}_{C_0}\tilde{F}_C(Z)$ with

$$\tilde{D}_{C_0} = \pm 2n_i^3 \sin^3\theta_i \cos^2\theta_i / k_0 n_r^2(n_i^2 - n_r^2)\{(1 + n_i^2/n_r^2)\sin^2\theta_i - 1\}$$

and

$$\tilde{F}_C(Z) = sh^2\gamma(sh^2\gamma + B_C)/(sh^2\gamma + C_\perp)(sh^2\gamma + C_\parallel)$$

with

$$B_C = 2(n_i^4 + n_r^4)\cos^2\theta_i(n_i^2\sin^2\theta_i - n_r^2)/(n_i^2 - n_r^2)^2$$
$$\{(n_i^2 + n_r^2)\sin^2\theta_i - n_r^2\}$$

The two signs of $\tilde{D}_{C_0}$ refer to left and right circular polarisation. The constants $C_\perp$ and $C_\parallel$ are given by (3.11) and (3.12).

6. *Discussion*

The study of the Goos–Hänchen effect on a layer can complete the information gathered studying the effect on a single totally reflecting plane. Along with a reflected beam a transmitted beam appears here and the dependence of their shifts on the layer thickness can be studied. It should be noted that in the case of a layer both beams are shifted even in the region of normal reflection ($\theta_i < \theta_c$) which is not the case with the reflection on a single plane.

The argument based on the energy flux within the layer seems more stringent and the results more general than the results of Section 3. However, for a layer this argument seems incomplete, yielding not the two shifts but only their linear combination. This is a consequence of translational invariance of plane waves. To overcome this difficulty one would have to use waveforms without translational invariance, e.g. the marked wave of Section 3. We extracted the information missing in the energy flux argument from the marked wave method, using equation (3.5).

As for the feasibility of experimental observation, it should be pointed out that layers with thickness of the order of magnitude of a wavelength are needed. Layers with $Z > \lambda_0$ are experimentally equivalent to a semi-infinite medium for most angles of incidence save the immediate neighbourhood of the critical angle. As, on the other hand, the magnitude of shifts increases with Z (Fig. 4), the region near the limit of observability of the transmitted beam would be optimal. It should also be emphasised that in the transmitted beam one actually measures $\tilde{D}_T$ and not $\tilde{D}'_T$ which was introduced for symmetry reasons.

In the visible range the longitudinal shift could possibly be observed by means of techniques described in the literature (Sandford, 1958; Coon, 1965; McDonald *et al.*, 1971). Either the transmitted or the reflected beam could be

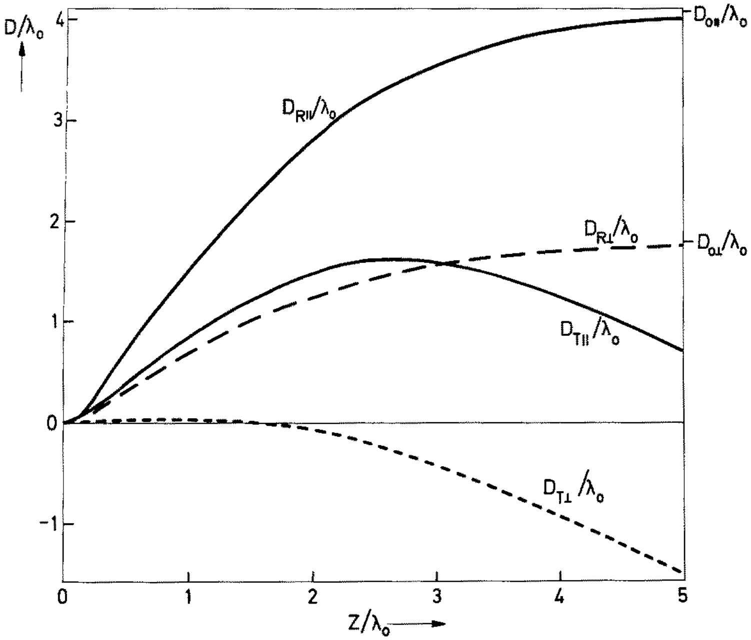

Figure 4.—Shifts $D_{R\perp}$, $D_{R\parallel}$ and $D_{T\parallel}$, $D_{T\parallel}$ for the particular case of Fig. 2 calibrated with $D_{0\perp}$ and $D_{0\parallel}$ as taken from the experimental data of Wolter (1950). In this case $D_{0\perp}$ coincides with $\tilde{D}_{0\perp}$ and $D_{0\parallel}$ coincides with $\tilde{D}_{0\parallel}$ if calculated according to equations (3.8), (3.9) and (4.6).

used and it seems that even the Z-dependence of the shifts might be measured if repeated reflections were used as in measuring the shift for a totally reflecting plane.

It would be tempting to perform the measurements with microwaves. Here the main problem is the beam width and divergence. Yet the shift of the order of a wavelength should be observable at a beam width, say, ten wavelengths even in a single reflection or transmission. In this case it would not be difficult to realise the layer as well-defined parallel gap of adjustable width.

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